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Evolution of the galaxy cluster temperature function

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Abstract. The evolution of the cluster temperature function in various density universes, using the mass density fluctuation field model which can explain the nearby cluster temperature function, has been examined. We have found that the amplitude of the cluster temperature function increases with increasing cluster redshift, while the amplitude of the cluster mass function is a monotonically decreasing function of the redshift at the same time. The increase in the amplitude is more significant for lower temperature clusters and in lower density universes in general. However, in a $\Omega_0 = 0.1$ universe, the increase of the amplitude is more significant in high temperature part up to a redshift of 2. These difference can be used as a good tool to discriminate between different density universes.

We predict that plenty of compact, high surface mass density clusters, exist at high redshifts in low density universes. Since the compactness of the clusters relates to the gravitational lensing power of the clusters, this difference between high and low density universe models can be distinguished using gravitational lensing statistics.

Key words: galaxies: clusters of – cosmology: theory – X-rays: galaxies

1. Introduction

The hierarchical gravitational clustering theory of the formation of structure in the universe has succeeded in explaining many observed features in the universe (e.g. Blumenthal et al. 1984). For example, the observational fact that a high fraction of nearby galaxy clusters have substructures (Forman & Jones 1990) is a strong evidence of clusters of galaxies growing by the agglomeration of smaller units. However, the fundamental parameters in the model, i.e. the mean density of the universe, the nature of the mass density fluctuation field in the early universe etc., are still unknown.

The clusters of galaxies have been recognized as a good tool for the measurement of these unknowns (e.g. Evrard 1989; Peebles et al. 1990; Edge et al. 1990; Henry & Arnaud 1991; Richstone et al. 1992; Oukbir & Blanchard 1992; Peebles 1993; Shimasaku 1993; White et al. 1993; Hattori 1994). Henry & Arnaud (1991) have constructed the complete cluster X-ray temperature function in the nearby universe and constrained the nature of the mass density fluctuation field in the early universe, i.e. the shape and amplitude of the density fluctuation power spectrum. They have shown that the X-ray temperature function works very well to constrain the nature of the mass density fluctuation field. Oukbir & Blanchard (1992), have found that there is a large difference of the expected number of the high redshift clusters between low and critical density universes and this large difference provides a good tool to discriminate between low and critical density universes.

Since the analysis of Henry & Arnaud (1991) was restricted to the case of critical density universe, in this paper we constrain the nature of the mass density field in a low density universe, by comparing the theoretical predictions and observational results of the temperature function of the nearby clusters constructed by Henry & Arnaud (1991). We show that it is difficult to explain the observed amplitude of the temperature function of nearby clusters by any model with $b \geq 1$ in low density universe, where b is the biasing parameter. Next we examine the evolution of the cluster temperature function in various densities universes using the mass density fluctuation field model, which can explain the temperature function of nearby clusters. This exercise has been attempted since Oukbir & Blanchard (1992) have only shown the difference of the integrated number of clusters up to some look back redshift. We have found that the evolutional nature of the cluster temperature function in different density universes are significantly different and it can be used as a good tool to discriminate between different density universes.

The plan of this paper is as follows. In Sect. 2, we explain the way to construct the theoretical cluster temperature function. In Sect. 3, the nature of the mass density fluctuation field in low density universes is constrained by comparing the theoretical predictions with the observational results of the temperature function of the nearby clusters. In Sect. 4, the evolution of the cluster temperature function is examined. In Sect. 5, we present our conclusions.

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2. Theoretical basis

In order to construct the temperature function, the mass function and the mass-temperature relation for virialized objects are needed. Here we follow the Press-Schechter (1974) approach, that is, we assume that the only objects which exist at a given epoch are those which have just collapsed and that any information about substructures is erased and smoothed out by the subsequent collapse. Peacock & Heavens (1990) have shown that the exact form of the mass function deduced using the above assumptions depends on the assumed filter functions, but in some cases it is exactly identical to the original Press-Schechter mass function. We use the Press-Schechter mass function (1974),

$$n(M,z)dM = \sqrt{\frac{2}{\pi}}\overline{\rho(z)}\frac{1}{M}\frac{\delta_z}{\sigma_M^2}\left(-\frac{d\sigma_M}{dM}\right)\mathrm{e}^{-\frac{\delta_z^2}{2\sigma_M^2}}\,dM, \quad (1)$$

where $\overline{\rho(z)}$ is the mean density of the universe at redshift z, δ_z is the density contrast, that a perturbation with this amplitude collapses at redshift z and σ_M is the standard deviation of the density contrast within randomly placed spherical windows which contains mass M at energy equipartition epoch, z_{eq}.

To get the temperature function, we have to transform the cluster mass to the cluster temperature in Eq. (1) and for this we require the mass-temperature relation. A mass-temperature relation is given in following way. Numerical experiments of the cluster formation (e.g. Evrard 1989) have shown that the temperature obtained from the relation $2\frac{3}{2}\frac{k_BT}{\mu m_{\mathrm{H}}} = f^2\frac{GM}{r_{\mathrm{max}}/2}$ is a good indicator of the virial temperature of the final states of the clusters, where r_{max} is the maximum expansion radius defined in the Appendix and f is a numerical constant with a best fit value of 1.1 (as shown by Evrard 1989). Using this, the following expression for the virial temperature of the cluster is obtained (see Eq. [19.5] in Peebles 1980; and also Appendix);

$$k_BT = k_BT_{8h^{-1}}(1+z_{\mathrm{eq}})\left(\frac{M}{M_{8h^{-1}}}\right)^{\frac{2}{3}}(\delta_z - \delta_c^*), \quad (2)$$

where Ω_0 is the ratio of the mean density of the universe to the density required to close the universe at present, $M_{8h^{-1}} = 5.9\times 10^{14}\Omega_0 h^{-1}M_\odot$ is a mass within a sphere of radius $8h^{-1}$Mpc in the present universe, δ_c^* is the threshold density contrast as defined in the Appendix, and $k_BT_{8h^{-1}} \equiv f^2\frac{2}{3}\mu m_{\mathrm{H}}\frac{GM_{8h^{-1}}}{8h^{-1}\mathrm{Mpc}} = 1.66\Omega_0$keV. Here we assume the spherical top-hat density perturbation model and a power law form of the power spectrum of the initial density perturbations with power index n for simplicity, i.e. $\sigma_M = \frac{5}{3}\frac{D(t(z_{\mathrm{eq}}))}{D(t_0)}\frac{\nu}{b}(\frac{M}{M_{8h^{-1}}})^{-\frac{n+3}{6}}$. The rms mass perturbation, $\sigma_M \equiv \delta M/M$, within a randomly-placed sphere containing mass M has been normalized such that, at the present epoch, $\delta M/M = 1/b$ at $8h^{-1}$Mpc where b is the biasing parameter. When $\Omega_0 = 1$ and $\delta_c^* = 0$, we see that Eq. (2) is exactly the same as Eq. (4) of Evrard (1989) but with a velocity dispersion as in Evrard (1989). In the open universe, δ_c^* has a positive finite value and it gives a threshold density contrast for the perturbation to collapse due to its self-gravity. Solving Eq. (2) for M and taking a partial derivative with respect to T, we get

$$M = [\frac{T}{T_{8h^{-1}}(1+z_{\mathrm{eq}})(\delta_z - \delta_c^*)}]^{\frac{3}{2}}M_{8h^{-1}}, \quad (3a)$$

and

$$\frac{\partial M}{\partial T} = \frac{3}{2}\frac{1}{T}[\frac{T}{T_{8h^{-1}}(1+z_{\mathrm{eq}})(\delta_z - \delta_c^*)}]^{\frac{3}{2}}M_{8h^{-1}}. \quad (3b)$$

Now we are ready for transforming variables and constructing a temperature function. By transforming the variables in Eq. (1) from M to k_BT using Eq. (3a) and (3b), the number density of the clusters at some redshift, z, with the temperature between k_BT and $k_BT + k_BdT$, (that is, temperature function), is given by

$$n(k_BT,z)k_BdT = n(M,z)\frac{\partial M}{\partial T}\,dT. \quad (4)$$

3. Mass density fluctuation field in low density universe

Henry & Arnaud (1991) have constrained the nature of the mass density fluctuation field in a critical density universe by using the temperature function of the nearby clusters. They have shown that the standard CDM spectra are incompatible with the observed X-ray temperature function. The best fit values are $n = -1.7$ and $b = 1.7$. This is shown in Fig. 2(a). In Fig. 1, the theoretical predictions of the differential temperature function in low density universes, that is $\Omega_0 = 0.2$ and $\Omega_0 = 0.1$, are compared with the observational results of nearby clusters obtained by Henry & Arnaud (1991). From Figs. 1a and 1b, it can be seen that it is difficult to get a reasonable fit to the observed temperature function by a $b \geq 1$ model in low density universes. In a $\Omega_0 = 0.2$ universe, none of the spectra with $b \geq 1$ are able to reproduce the observed amplitude of the low temperature part of the temperature function. Their predicted amplitudes are always less than the observed one. In a $\Omega_0 = 0.1$ universe, the difference between $b \geq 1$ model predictions and the observed temperature function is even more significant than the $\Omega_0 = 0.2$ case. One of the reasonable fitting models for each Ω_0 is also shown in Fig. 1. Adopted models are $n = -0.9$ and $b = 0.7$ for $\Omega_0 = 0.2$ and $n = -0.5$ and $b = 0.4$ for $\Omega_0 = 0.1$. The range of the temperature where the above results can be safely applied is 3 – 10keV. This is interpreted as the mass range $0.3 - 2\times 10^{15}h^{-1}\mathrm{M}_\odot$ (see Fig. 3).

Recently Henry et al. (1994) have performed the first measurement of the ensemble properties of X-ray selected groups of galaxies and have obtained the amplitude of the temperature function at $T = 1.4$keV. The comparison between this result and the predictions of the adopted models is very interesting and gives us a chance to measure the mass density fluctuation spectrum for a larger mass range. The critical density universe model ($\Omega_0 = 1,\ n = -1.7,\ b = 1.7$) can also fit the temperature function of the groups of galaxies. On the other hand, the predicted number density of low density universe models is less than the observed value. We could not find any single power law power spectrum model in low density universes which can

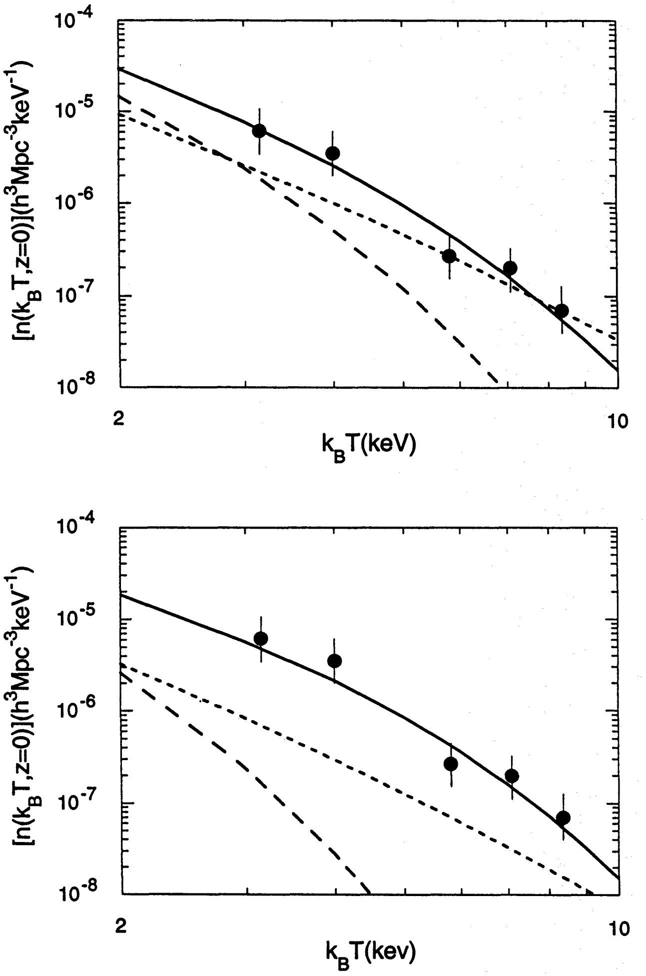

Fig. 1a and b. The comparison of the theoretical cluster temperature function with observations (filled circles; taken from Henry & Arnaud 1991). The upper solid line shows the theoretical prediction of the present temperature function of one of the mass density fluctuation field models which can give reasonably good fit to the observations, (**a**) $n = -0.9$, $b = 0.7$ in $\Omega_0 = 0.2$, (**b**) $n = -0.5$, $b = 0.4$ in $\Omega_0 = 0.1$. As a comparison, the theoretical predictions with $b = 1$ models are also shown, short dashed lines for $n = -2$ and long dashed lines for $n = -1$

reproduce Henry et al (1994)'s temperature function for the full range of the temperatures. However, we note that it is still not clear whether the temperatures of the groups of galaxies are really tracing the virial temperature of the cluster potential as in the case of rich clusters (Lubin & Bahcall 1993) or it is significantly heated by the energy ejection from the member galaxies. The latter is more likely at present, since the hot gas distribution in the groups is much flatter than that of rich clusters (e.g. Henry et al. 1994). If this is the case, the virial temperature of the groups is lower than the observed temperature and there is still a possibility that the low density universe models adopted here can reproduce the full range of the observed temperature function. The study of the galaxy distribution and the much more accurate measurement of the velocity dispersion of galaxies in each groups are necessary before we can constrain the cosmological model using the temperature function of the groups of galaxies. In the following study, we assume that the single power law model of the power spectrum of the mass density fluctuation field is applicable to the temperature range from 0.5keV to 30keV, that is corresponding to the mass range of $10^{13}h^{-1}\mathrm{M}_\odot \leq M \leq 10^{16}h^{-1}\mathrm{M}_\odot$.

4. The evolution of the cluster temperature function

We next examine the evolution of the cluster temperature function in various density universes using the mass density fluctuation field model, which can explain the nearby cluster temperature function. The models which best fit the nearby cluster temperature function, as shown in Sec.2, are adopted, that is $n = -1.7$, $b = 1.7$ in $\Omega_0 = 1$, $n = -0.9$, $b = 0.7$ in $\Omega_0 = 0.2$ and $n = -0.5$, $b = 0.4$ in $\Omega_0 = 0.1$. Figure 2 shows the evolution of the cluster temperature function of these models. Hereafter, we refer to $n(k_BT, z)/(1+z)^3$ as the cluster temperature function. The amplitude of the temperature function in any density universe is not a simple monotonically decreasing function of redshift. In some low temperature range, the amplitude increases and then starts to decrease from some redshift. The increase of the amplitude is more significant for lower temperature clusters and in lower density universes in general. However, in a $\Omega_0 = 0.1$ universe, the increase of the amplitude is more significant in the higher temperature part, upto a redshift 2. These differences can be used as a good tool to discriminate between different density universes. The amplitude of the cluster mass function is, on the other hand, a monotonically decreasing function of redshift. The change of temperature and amplitude of the temperature function with redshift, for fixed mass clusters, are indicated by dashed lines for $M = 10^{13}\mathrm{M}_\odot$ and dashed-dotted lines for $M = 10^{14}\mathrm{M}_\odot$ in Fig. 2. It shows that the temperature and the amplitude of temperature function of same mass clusters at higher redshift is higher and smaller, respectively. Since the cluster mass function is proportional to the product of the temperature function and the inverse square of the temperature (see Eqs.(3b) and (4)), it means that the amplitude of cluster mass function is a monotonically decreasing function of redshift in any density universe.

The above mentioned results, that the amplitude increases once and starts to decrease from some redshift, show that there is a redshift at which the cluster number density, for a fixed temperature cluster, gets maximum. Hereafter we shall call this redshift as the maximum number density epoch, z_{mxn}. Figure 3 shows the maximum number density epoch for our adopted models. We see that z_{mxn} is higher for lower temperature clusters and in lower density universes. In low density universes, the maximum number density epoch is earlier than present, that is $z_{\mathrm{mxn}} > 0$, for all temperatures in the range $T \leq 30$keV. On the other hand, in a critical density universe, only for very low temperature clusters, that is for $T \leq 1$keV, the maximum number density epoch appears earlier than present. Clusters with $T > 1$keV have not yet met the maximum number density epoch and the number density of these clusters is still increasing. The maximum number density epoch depends not only on the

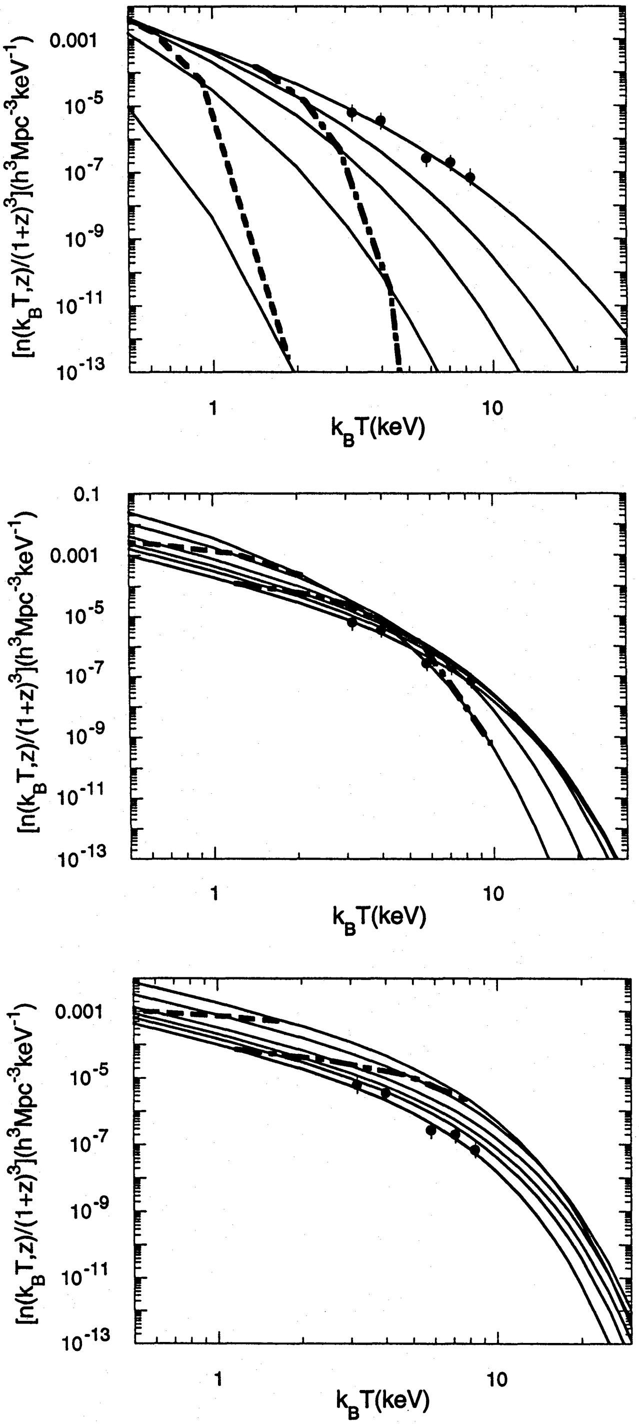

Fig. 2a–c. The evolution of the cluster temperature function using the mass density fluctuation field model which can explain the nearby cluster temperature function is shown, **a** $n = -1.7$, $b = 1.7$ in $\Omega_0 = 1$ (best fitting model of Henry & Arnaud 1991) and from right to left at the bottom of the figure $z = 0,\ 0.5\ ,1\ ,2\ ,5$, **b** $n = -0.9$, $b = 0.7$ in $\Omega_0 = 0.2$ and from bottom to up at the left corner of the figure $z = 0,\ 0.5\ ,1\ ,2\ ,5,\ 10$, **c** $n = -0.5$, $b = 0.4$ in $\Omega_0 = 0.1$ and from bottom to up at the left corner of the figure $z = 0,\ 0.5\ ,1\ ,2\ ,5,\ 10$. The change of temperature and amplitude of the temperature function with redshift for fixed mass clusters are indicated by dashed lines for $M = 10^{13}M_\odot$ and dashed-dotted lines for $M = 10^{14}M_\odot$ when h=1

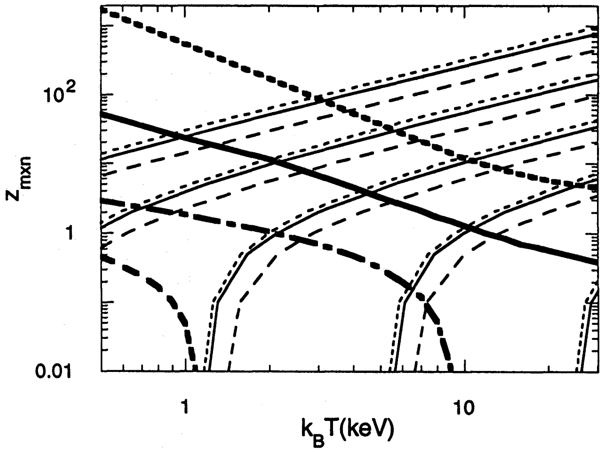

Fig. 3. The maximum number density epoch versus cluster temperature is shown, bold long dashed line for $n = -1.7$, $b = 1.7$ in $\Omega_0 = 1$, bold solid line for $n = -0.9$, $b = 0.7$ in $\Omega_0 = 0.2$, bold short dashed line for $n = -0.5$, $b = 0.4$ in $\Omega_0 = 0.1$ and bold dashed dotted line for $n = -2$, $b = 1$ in $\Omega_0 = 0.2$. Constant cluster mass contours are also drawn, long dashed lines for $\Omega_0 = 1$, solid lines for $\Omega_0 = 0.2$ and short dashed lines for $\Omega_0 = 0.1$. From left to right, $10^{12}M_\odot$, $10^{13}M_\odot$, $10^{14}M_\odot$, $10^{15}M_\odot$ and $10^{16}M_\odot$, respectively. Here h=1 is assumed

density of the universe but also on the nature of the mass density fluctuation field in the early universe. A dashed-dotted line in Fig. 3 shows z_{mxn} for $n = -2$, $b = 1$ in $\Omega_0 = 0.2$ and how it depends on the nature of the mass density fluctuation field. Higher biasing and redder spectra results in smaller z_{mxn}.

The existence of plenty of compact clusters at high redshift is expected in a low density universe as pointed out by the previous studies (e.g. Richstone et al. 1992). Constant mass contours are drawn in Fig. 3. These lines do not depend on the nature of the mass density field and just depend on Ω_0 but the dependence is very small. These lines show that higher temperature clusters are formed from the same mass at higher redshift. This is because the high redshift clusters are more compact than the present clusters and the cluster virial temperature is not only proportional to the mass of the cluster but is also proportional to the inverse of the radial scale of the cluster. This is true for both high and low density universes. However, for such high redshift compact clusters, there is a significant difference in the number density for high and low density universes. Since in low density universes the maximum number density epoch is earlier than present in cluster temperature range, existence of plenty of compact clusters in high redshift is expected in low density universes. On the other hand, in a critical density universe, since the amplitude of temperature function decreases rapidly with redshift, the number of such compact high redshift clusters is expected to be a small fraction of the nearby clusters' number. Since the compactness, in another words, the surface mass density of the clusters, relates to the gravitational lensing power, that is, a higher surface mass density cluster has a stronger gravitational lensing power (e.g Turner et al. 1984), this difference between high and low density universe models could be distinguishable by gravitational lensing statistics.

5. Conclusions

We constrain the nature of the mass density fluctuation field in a low density universe by comparing the theoretical predictions and observational results of the temperature function of the nearby clusters as constructed by Henry & Arnaud (1991). We have shown that it is difficult to get a reasonable fit to the observed temperature function by $b \geq 1$ models in a low density universe. Next, the evolution of the cluster temperature function, in various density universes, using the mass density fluctuation field model, which can explain the nearby cluster temperature function, has been examined. We find that there is an increase in the amplitude of the cluster temperature function with increasing cluster redshift although the amplitude of the cluster mass function is a monotonically decreasing function of the redshift at the same time. This has also been found by Kaiser (1991), who has examined the evolution of the cluster temperature function for the critical density universe. We have shown that the increase of the amplitude is more significant for lower temperature clusters and in lower density universes in general. However, in $\Omega_0 = 0.1$ universe, the increase of the amplitude is more significant in the high temperature part, upto a redshift of 2. These differences can be used as a good tool to discriminate between different density universes. We have also shown that there is a redshift at which the cluster number density, for a fixed cluster temperature, becomes maximum. The maximum number density epoch is higher for lower temperature clusters and in lower density universes. We have also predicted that plenty of compact, high surface mass density clusters, exist at high redshift in low density universes. Since the compactness of clusters relates to the gravitational lensing power, this difference between high and low density universe models could be distinguishable by gravitational lensing statistics.

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Appendix A

The linear extrapolated present day density contrast of the perturbation which collapses at redshift z_c in an open universe is presented. Suppose there exists, at some early epoch z_i, from when the density perturbation starts to grow, a spherical region with radius r_i which has uniform density, ρ_i, slightly higher than the mean density of the universe, $\overline{\rho_i}$, and includes a mass, M. z_i is the equipartition epoch in the non-baryonic dark matter dominant universe, say $1 + z_i = 1 + z_{eq} = 2.4 \times 10^4 \Omega_0 h^2$. Development of the outer most radius of this region is described by the equation of the energy conservation as

$$\frac{1}{2}(\frac{dr}{dt})^2 - \frac{GM}{r} = \frac{1}{2}H_i^2 r_i^2 - \frac{GM}{r_i}, \qquad (A1)$$

where $H_i = H_0\sqrt{\Omega_0(1+z_i)^3 + (1+z_i)^2(1-\Omega_0)}$ is the Hubble constant at redshift z_i. Using the maximum expansion radius, r_{max}, Eq. (A1) is rewritten as

$$\frac{1}{2}(\frac{dr}{dt})^2 - \frac{GM}{r} = -\frac{GM}{r_{max}}. \qquad (A2)$$

By integrating this equation, the time when this region eventually collapse is calculated as

$$t_c = \pi\sqrt{\frac{r_{max}^3}{2GM}}. \qquad (A3)$$

In the low density universe, there is a threshold density contrast for the density perturbation to collapse due to its self gravity (Peebles 1980) described by,

$$\delta_c^* = \frac{\rho_{cr,i} - \overline{\rho_i}}{\overline{\rho_i}}, \qquad (A4)$$

where $\rho_{cr,i} = 3H_i^2/8\pi G$ is the critical density at redshift z_i. The density contrast of the perturbed region, $\delta_i^* = (\rho_i - \overline{\rho_i})/\overline{\rho_i}$, is expressed by the sum of δ_c^* and $\widetilde{\delta_i} = (\rho_i - \rho_{cr,i})/\overline{\rho_i}$. By using these notations, the right hand side of Eq. (A1) is rewritten as

$$\frac{1}{2}H_i^2 r_i^2 - \frac{GM}{r_i} \sim -\frac{GM}{r_i}\widetilde{\delta_i}, \qquad (A5)$$

where second orders terms in δ are neglected. From Eqs.(A2) and (A5), the expression for the maximum expansion radius is obtained as

$$r_{max} = \frac{r_i}{\widetilde{\delta_i}} = \frac{r_i}{\delta_i^* - \delta_c^*}. \qquad (A6)$$

By inserting Eq. (A6) into Eq. (A3), the expression for the collapse time is obtained as

$$t_c = \frac{\pi}{H_i}\Omega_i^{-1/2}\widetilde{\delta_i}^{-3/2}, \qquad (A7)$$

where $\Omega_i = \overline{\rho_i}/\rho_{cr,i}$ is density parameter at redshift z_i. Therefore, to collapse at t_c the density perturbation must have the density excess to the mean density given by

$$\delta_i^* = \delta_c^* + (\frac{\pi}{H_i t_c})^{2/3}\Omega_i^{-1/3}. \qquad (A8)$$

By using the following relation, the collapse time is transformed into the collapse redshift, z_c;

$$t_c = \frac{1}{H_c}[\frac{1}{1-\Omega_c} - \frac{\Omega_c}{2(1-\Omega_c)^{3/2}}\cosh^{-1}(\frac{2}{\Omega_c} - 1)], \qquad (A9)$$

where the suffix c specifies the physical variables at the collapse redshift. The amplitude of the perturbation which has collapsed at redshift z_c is given by

$$\delta_{z_c} = [\delta_c^* + (\frac{\pi}{H_i t_c})^{2/3}\Omega_i^{-1/3}], \qquad (A10)$$

and then, the linear extrapolated present day amplitude is given by

$$\frac{3}{5}\frac{D(t_0)}{D(t(z_i))}\delta_{z_c}$$

and

$$D(t(z)) = 1 + \frac{3}{x} + \frac{3(1+x)^{1/2}}{x^{3/2}} \ln[(1+x)^{1/2} - x^{1/2}], \quad (A11)$$

where $D(t(z))$ is a linear growth factor of the density perturbation (Peebles 1980), $x = (\Omega_0^{-1} - 1)/(1+z)$, the factor $3/5$ comes from the fact that the amplitude of the growing mode is $\frac{3}{5}\delta_i^*$ and t_0 is age of the universe. In the limit of $\Omega_0 \rightarrow 1$, $[D(t_0)/D(t(z_i))] \rightarrow (1+z_i) = (t_0/t_i)^{2/3}$, $\frac{1}{H_i t_i} \rightarrow \frac{3}{2}$, $\Omega_i \rightarrow 1$ and $\delta_c^* \rightarrow 0$. Then, the linear extrapolated present day amplitude of the perturbation which collapses at present is given by,

$$\frac{3}{5}\frac{D(t_0)}{D(t(z_i))}\delta_{\mathrm{cr},0} \rightarrow \frac{3}{5}(\frac{3}{2}\pi)^{2/3} \sim 1.69, \quad (A12)$$

in the limit of $\Omega_0 \rightarrow 1$. Therefore our result reproduces the famous result obtained in a $\Omega_0 = 1$ universe (e.g. Peebles 1980) in the limit of $\Omega_0 \rightarrow 1$.

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